A Multidimensional System for Compositional Dynamic Semantics

This paper proposes a compositional dynamic semantics with a **static semantic value** $[\![\cdot]\!]^g$ almost identical to Heim and Kratzer (1998) and a **dynamic semantic value** $\langle\!\langle\cdot\rangle\!\rangle$ like Dynamic Predicate Logic (DPL, Groenendijk and Stokhof 1991). DPL with selective generalized quantifiers (see Van Eijck and De Vries 1992, Chierchia 1992, 1995) is replicated without "generalizing to the worst case" as in Muskens (1996). The left-to-right order of dynamic updates is fixed using a single rule, rather each lexical entry encoding this order, as in Muskens' system.

The new system treats matrix sentences (among other nodes) as dynamic "tests" via the **dynamic lift operator** defined in (1). For instance, a matrix sentence like A^w *woman entered* is first assigned a dynamic meaning via the $\langle\!\langle\cdot\rangle\!\rangle$ function, to be defined below (e.g., this value includes random assignment of the variable w.) Next, the lift operator filters out assignments g where the sentence is false, ensuring that each $g(w)$ is a woman who entered. A version for properties α is also given: ${}^\wedge\alpha(v)$ filters out any g making $[\![\alpha]\!]^g(g(v))$ false.

(1) Dynamic Lift: For node α and variable v, where '$\circ$' is relation composition
$${}^\wedge\alpha = \langle\!\langle\alpha\rangle\!\rangle \circ \left\{\langle g, g\rangle : [\![\alpha]\!]^g{=}1\right\}, \qquad {}^\wedge\alpha(v) = \langle\!\langle\alpha\rangle\!\rangle \circ \left\{\langle g, g\rangle : [\![\alpha]\!]^g(g(v)){=}1\right\}$$

Next, (2) defines two special cases (more are defined below for generalized quantifiers). When D is indefinite, $[\![\mathrm{D}^v\beta]\!]^g$ is simply the individual $g(v)$, but $\langle\!\langle\mathrm{D}^v\beta\rangle\!\rangle$ has two effects: the random assignment of the variable v composed with the dynamic lift ${}^\wedge\beta(v)$. For instance, $[\![\mathbf{a}^w\ \mathbf{woman}]\!]^g$ is $g(w)$ and $\langle\!\langle\mathbf{a}^w\ \mathbf{woman}\rangle\!\rangle$ introduces the variable w and then tests that w is a woman, equivalent to the DPL formula $\boldsymbol{[w]; woman(w)}$. As for negated clauses, $[\![\textsc{Neg}\ \beta]\!]^g$ is only true when there is no way to satisfy the dynamic lift ${}^\wedge\beta$ given g as the input assignment. For instance $[\![$John doesn't own a^c car$]\!]^g{=}1$ iff there's no h such that $h(c)$ is a car that John owns, equivalent to the DPL formula $\boldsymbol{\neg([c]; car(c); owns(j, c))}$. Negation clear any dynamic effects below it, and therefore $\langle\!\langle\textsc{Neg}\ \beta\rangle\!\rangle$ is the trivial test/identity relation $\top = \{\langle g, h\rangle : g{=}h\}$.

(2) Special Cases

a. **Indefinite DPs** $\quad [_\alpha\ \mathrm{D}^v\ \beta] : [\![\alpha]\!]^g{=}g(v) \qquad$ and $\langle\!\langle\alpha\rangle\!\rangle = \{\langle g, h\rangle : g[v]h\} \circ {}^\wedge\beta(v)$

b. **Negated Clauses** $\quad [_\alpha\ \textsc{Neg}\ \beta] : [\![\alpha]\!]^g{=}\begin{cases} 0, \text{ if } \exists h\,\left(g\ ({}^\wedge\beta)\ h\right) \\ 1, \text{ otherwise}\end{cases} \qquad$ and $\langle\!\langle\alpha\rangle\!\rangle = \top$

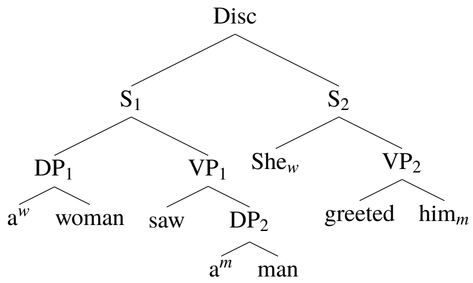
Remaining static semantic values are precisely as defined in Heim and Kratzer (1998) and remaining dynamic semantic values are as in (3): a terminal node's dynamic value is trivial, and a branching node's value is the left-to-right composition of its children's dynamic values. The dynamic value of a discourse sequence of one or more sentences is the composition of these component sentences' dynamic lifts (i.e., the components act as tests).

(3) Dynamic Semantic Value Apart from special cases,

Terminal Nodes α: $\langle\!\langle\alpha\rangle\!\rangle = \top \qquad$ **Branching Nodes** $[_\alpha\ \beta\ \gamma] : \langle\!\langle\alpha\rangle\!\rangle = \langle\!\langle\beta\rangle\!\rangle \circ \langle\!\langle\gamma\rangle\!\rangle$

Discourse Sequences $[_\alpha\ \sigma_1 \ldots \sigma_n]$: $\langle\!\langle\alpha\rangle\!\rangle = {}^\wedge\sigma_1 \circ \ldots \circ {}^\wedge\sigma_n$, when $n \geq 1$

Figure 1 is a sample calculation (numerical indices are for reference only, and dynamic values are shown as DPL formulas). Denotations for DP_1 and DP_2 are calculated via the Indefinite DPs rule. $[\![\mathrm{VP}_1]\!]^g$ is calculated via Functional Application and $\langle\!\langle\mathrm{VP}_1\rangle\!\rangle$ via the Branching Nodes rule; since $\langle\!\langle$saw$\rangle\!\rangle$ is trivial, $\langle\!\langle\mathrm{VP}_1\rangle\!\rangle{=}\langle\!\langle\mathrm{DP}_1\rangle\!\rangle$. $[\![\mathrm{S}_1]\!]^g$ is standard, and $\langle\!\langle\mathrm{S}_1\rangle\!\rangle$ randomly assigns w and m and limits them to women and men respectively. ${}^\wedge\mathrm{S}_1$ is the same as $\langle\!\langle\mathrm{S}_1\rangle\!\rangle$, except it



$[\![\mathrm{DP}_2]\!]^g = g(m)$ $\langle\!\langle \mathrm{DP}_2 \rangle\!\rangle = \boldsymbol{[m]; man(m)}$

$[\![\mathrm{saw}]\!]^g = \lambda y.\lambda x.\mathbf{saw}(x, y)$ $\langle\!\langle \mathrm{saw} \rangle\!\rangle = \top$

$[\![\mathrm{VP}_1]\!]^g = \lambda x.\mathbf{saw}(x, g(m))$ $\langle\!\langle \mathrm{VP}_1 \rangle\!\rangle = \boldsymbol{[m]; man(m)}$

$[\![\mathrm{DP}_1]\!]^g = g(w)$ $\langle\!\langle \mathrm{DP}_1 \rangle\!\rangle = \boldsymbol{[w]; woman(w)}$

$[\![\mathrm{S}_1]\!]^g = 1$ iff $\mathbf{saw}(g(w), g(m))$

$\langle\!\langle \mathrm{S}_1 \rangle\!\rangle = \boldsymbol{[w]; woman(w); [m]; man(m)}$

${}^\wedge\mathrm{S}_1 = \boldsymbol{[w]; woman(w); [m]; man(m); saw(w,m)}$

$[\![\mathrm{S}_2]\!]^g = 1$ iff $\mathbf{greeted}(g(w), g(m))$ $\langle\!\langle \mathrm{S}_2 \rangle\!\rangle = \top$

${}^\wedge\mathrm{S}_2 = \boldsymbol{greeted(w,m)}$ $\langle\!\langle \mathrm{Disc} \rangle\!\rangle = {}^\wedge\mathrm{S}_1 \circ {}^\wedge\mathrm{S}_2$, i.e.. . .

$\langle\!\langle \mathrm{Disc} \rangle\!\rangle = \boldsymbol{[w]; woman(w); [m]; \ldots greeted(w,m)}$

Figure 1: Sample calculation of a short discourse

also tests that the woman w saw the man m. $[\![\mathrm{S}_2]\!]^g$ is standard, too, but $\langle\!\langle \mathrm{S}_2 \rangle\!\rangle$ is trivial (since S_2 contains no dynamic elements). ${}^\wedge\mathrm{S}_2$ thus simply tests whether w greeted m. The whole discourse has a dynamic meaning which is the composition of the dynamic lifts of S_1 and S_2.

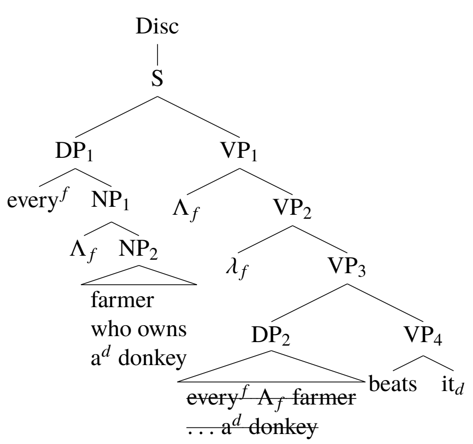
Next, following the copy theory of movement (Chomsky 1995), a generalized quantifier like DP_1 in Figure 2 raises, leaving a copy behind (DP_2 here) and a the λ_f operator. Notice, too, that both NP_1 and VP_1 include a dynamic abstractor Λ_f. Two more special cases are thus defined in (4). **Trace Conversion** (applying to lower-copy DPs) is identical to the Indefinite DPs rule except without random assignment. This means the dynamic effects of a restrictor like NP_1 are reintroduced via the lower copy. **Dynamic Abstraction** incorporates dynamic meanings into a static property, essentially true of any individual that satisfies the dynamic meaning of its complement when replacing v. (The dynamic value of Dynamic Abstraction is trivial.)

(4) Additional Special Cases

a. **Trace Conversion** $[_\alpha\ \mathrm{D}^v\ \beta]$: $[\![\alpha]\!]^g = g(v)$ and $\langle\!\langle \alpha \rangle\!\rangle = {}^\wedge\beta(v)$.

b. **Dynamic Abstraction** $[_\alpha\ \Lambda_v\ \beta]$: $[\![\alpha]\!]^g = \lambda x.\exists h \left(g^{x/v} \left({}^\wedge\beta(v) \right) h \right)$ and $\langle\!\langle \alpha \rangle\!\rangle = \top$.

Figure 2 shows a sample quantifier calculation. Lower-copy DP_2 makes its largest contribution to the dynamic (rather static) value of the VP, essentially binding the pronoun it_d. $[\![\mathrm{NP}_1]\!]^g$ and $[\![\mathrm{VP}_1]\!]^g$ denote properties true of farmers satisfying their dynamic values, appropriate inputs to a standard quantifier meaning for *every*. Finally, the discourse uses the sentence as a test. (Note this derives the weak donkey reading; the full paper addresses the strong reading as well.)



$[\![\mathrm{NP}_2]\!]^g = \lambda x.\mathbf{farmer}(x)\ \&\ \mathbf{owns}(x, d)$

$\langle\!\langle \mathrm{NP}_2 \rangle\!\rangle = \boldsymbol{[d]; donkey(d)}$

${}^\wedge\mathrm{NP}_2(f) = \boldsymbol{[d]; donkey(d); farmer(f); owns(f,d)}$

$[\![\mathrm{NP}_1]\!]^g = \lambda x.\exists h \left(g^{x/f} \left({}^\wedge\mathrm{NP}_2(f) \right) h \right)$

$= \lambda x.\exists d(\mathbf{donkey}(d)\ \&\ \mathbf{farmer}(x)\ \&\ \mathbf{owns}(x, d))$

$[\![\mathrm{DP}_2]\!]^g = g(f)$ $\langle\!\langle \mathrm{DP}_2 \rangle\!\rangle = \langle\!\langle \mathrm{VP}_3 \rangle\!\rangle = \langle\!\langle \mathrm{VP}_2 \rangle\!\rangle = {}^\wedge\mathrm{NP}_2(f)$

$[\![\mathrm{VP}_2]\!]^g = \lambda f.\mathbf{beats}(f, g(d))$, ${}^\wedge\mathrm{VP}_2(f) = \langle\!\langle \mathrm{VP}_2 \rangle\!\rangle \circ \boldsymbol{beats(f,d)}$

$[\![\mathrm{VP}_1]\!]^g = \lambda x.\exists d \left(\begin{array}{l} \mathbf{donkey}(d)\ \&\ \mathbf{farmer}(x)\ \& \\ \mathbf{owns}(x, d)\ \&\ \mathbf{beats}(x, d) \end{array} \right)$

$[\![\mathrm{S}]\!]^g = 1$ iff for all x s.t. $[\![\mathrm{NP}_1]\!]^g(x) = 1$, $[\![\mathrm{VP}_1]\!]^g(x) = 1$

$\langle\!\langle \mathrm{S} \rangle\!\rangle = \top$ $\langle\!\langle \mathrm{Disc} \rangle\!\rangle = {}^\wedge\mathrm{S} = \{\langle g, g \rangle : [\![\mathrm{S}]\!]^g = 1\}$

Figure 2: Sample calculation of a donkey sentence